Hall Ticket Number:

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VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CSE: CBCS) III-Semester Backlog (Old) Examinations, December-2018

Discrete Structures

Time: 3 hours Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (10 × 2 = 20 Marks)

1. If p and q are false, is $((\sim p)\vee q) \wedge (p\vee(\sim q))$ true or false ?
   If p and q are false, is $(p\vee q) \wedge (p\vee(\sim q))$ true or false ?
2. Show that $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$
3. Find GCD of 128,248.
4. Prove that composition of two one-one functions is one-one.
5. What is the generating function for the series 0,2,6,12.........
6. There are n guests at a party. Each person shakes hand with everybody else exactly 2 times. Define the number of handshakes that occur recursively.
7. Consider the ring Z_{12} = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11) of integers modulo 12. Find the Units of Z_{12}.
8. Use Fermat's theorem to compute 3^{302} (mod 11).
9. Let C be a set of code words where $C \subseteq Z_2^{\,7}$. If the error pattern e = 0101111 and the received word r = 0100111 are given find the code word C.
10. Find a subgroup of order 2 of the group Z_8.

Part-B (5 × 10 = 50 Marks)

11. a) Prove the following : (clearly write the steps and reasons) [5]

$$\forall x\, [p(x)] \vee q(x)]$$
$$\forall x\, [((\neg p(x)) \vee q(x))) \longrightarrow r(x)$$
$$\overline{}$$
$$\therefore \forall x\, [\neg r(x)) \longrightarrow p(x)\,]$$

b) Prove that $u \rightarrow r, (r \wedge s) \rightarrow (p \vee t), q \rightarrow (u \wedge s), \neg t \;\; \therefore q \rightarrow p$ using conditional proof. [5]

12. a) A company hires 11 employees each of whom is to be assigned to one of four subdivisions. Each sub division will get at least one new employee. In how many ways can these assignment be made? [5]

b) Determine whether the following binary functions are commutative and associative.

i) f(x, y)= max {x, y} (that is maximum of x and y)

ii) $g(x, y) = x^y$

13. a) Solve $F_n=F_{n-1}+F_{n-2}$ where $F_0=0,F_1=1$ [5]

b) There are four colors poker chips –red, white, green and blue. Find and solve the recurrence relation for the number of ways to stack n of these poker chips so that there are no consecutive blue chips. [5]

Contd... 2

14. a) Let (R, +, .) and (S, ⊕, ⊙) be rings with zero elements z_R and z_S respectively. Let f : R ⟶ S be a ring homomorphism. Let K = {a ∈ R: f(a) = z_S }. Prove that f is one-one if and only if K = {z_R}. Using this fact, verify whether the ring homomorphism f: Z ⟶ Z_6 defined by f(x) = $[x]_6$ is one-one or not. [7]

b) Prove or disprove in a ring R if a, b are units of R then ab is a unit of R. [3]

15. a) Let G be group of order n and H a subgroup of order m. Define the relation R on G by for a, b ∈ G, aRb if $a^{-1}b$ ∈ H. Prove that R is an equivalence relation. Prove also that if a ∈ G, then [a], the equivalence class of a, which is defined as {b ∈ G : bRa}, satisfies [a] = aH. [7]

b) When x, y ∈ $Z_2^{\ n}$ we define d(x, y) = $\sum_{i=1}^{n} \bar{d}(x_i,\ y_i)$ where $\bar{d}(x_i,\ y_i) = \begin{cases} 0 \ if \ x_i = y_i \\ 1 \ if \ x_i \neq y_i \end{cases}$. Prove that for all x, y, z ∈ $Z_2^{\ n}$, d(x, z) ≤ d(x, y) + d(y, z) [3]

16. a) Apply Mathematical induction to verify $\sum_{i=1}^{n} i(2^i)=2+(n-1)2^{n+1}$ [5]

b) f:A->B, g:B->C are bijectives prove that gof and f-1 is also bijective. [5]

17. Answer any *two* of the following:

a) Find coefficient of x^8 in the series $\dfrac{1}{(1-2x)^2(1-3x)}$ [5]

b) Find the minimum value of X which satisfies the following simultaneous equations X ≡ 1 (mod 5) X ≡ 2 (mod 6) X ≡ 3 (mod 7) [5]

c) The encoding function E : $Z_2^{\ 2} \rightarrow Z_2^{\ 2}$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Determine all the code words. What can you say about its error detection capability? [5]

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